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Supplement of

Bayesian hierarchical modelling of intensity-duration-frequency curves using a climate model large ensemble

Alexander Lee Rischmuller et al.

Correspondence to: Alexander Lee Rischmuller (alexander.rischmueller@uni-hamburg.de)

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1 Supplementary Methods

1.1 Linear interpolation dGEV-BHM-ξ_d

For the dGEV-BHM-ξ_d, linear interpolation is required to approximate the IDF curve.

1. For the y-values of the IDF plot, we compute a longer vector of durations by linearly interpolating between the discrete durations. Let the original set of durations from the resolution of the model be $\mathbf{d} = (d_1, d_2, \ldots, d_n)$. In our case, $\mathbf{d} = (1, 3, 6, 12, 24, 48)$ and, therefore, $n = 6$. The objective is to generate a new vector D by linear interpolation between each consecutive pair of elements in $\mathbf{d}$. Let m be the number of points between each pair (in our case $m = 99$). For each interval $[d_i, d_{i+1}]$, where $i = 1, 2, ..., n-1$, we generate m new points. Let these interpolated points be denoted by $d_{i,k}$, where $k = 1, 2, ..., m$. These points can be defined as:

$$d_{i,k} = d_i + k \times \frac{d_{i+1} - d_i}{m} \tag{S1}$$

   In our case, the total amount of elements in D will be 501.

2. As there are estimates of the shape parameter for each discrete durations we have to apply linear interpolation methods to estimate the IDF curve. In the first interpolation stage, we linearly interpolate between the mean shape parameter estimates $\bar{\xi}_d = \left(\bar{\xi}_1, \bar{\xi}_2, \ldots, \bar{\xi}_n\right)$ of the dGEV-BHM-ξ_d in the same way as we did with with duration vector described in Equation S1.

$$\bar{\xi}_{i,k} = \bar{\xi}_i + k \times \frac{\bar{\xi}_{i+1} + \bar{\xi}_i}{m} \quad \text{for } k = 1, 2, \ldots, m \tag{S2}$$

   Similar to the duration vector described earlier, with $m = 99$, the new vector denoted Ξ will also consist of 501 points.

3. Now we can utilize both D and Ξ to compute a preliminary IDF curve with confidence intervals using MCMC posterior parameter estimates for the other spatial parameters $\tilde{\mu}_j$, $\sigma_{0,j}$, θ_j, and η_j specific to the locality in question.

$$I_\Xi(D,T) = \begin{cases} \sigma_0(D+\theta)^{-\eta}\left(\tilde{\mu} + \frac{1}{\Xi}\left[\left(-\log\left(1-\frac{1}{T}\right)\right)^{-\Xi} - 1\right]\right), & \Xi \neq 0, \\ \sigma_0(D+\theta)^{-\eta}\left(\tilde{\mu} - \log\left(-\log\left(1-\frac{1}{T}\right)\right)\right), & \Xi = 0. \end{cases} \tag{S3}$$

   From this, we obtain a two-dimensional array of different IDF curves corresponding to each parameter estimate, from which we can derive the mean values and the corresponding confidence intervals.

4. Finally, the values of the preliminary IDF curve $I_\Xi(D,T)$ will not match the actual return levels at the discrete durations $\mathbf{d}$, as they are computed with model shape parameter $\xi_{\mathbf{d}}$. Hence, we will need to interpolate $I_\Xi(D,T)$ using the real return level estimates. First, we compute the return levels at discrete durations $\mathbf{d}$ using $\xi_{\mathbf{d}}$.

$$I(\mathbf{d},T) = \begin{cases} \sigma_0(\mathbf{d}+\theta)^{-\eta}\left(\mu_t + \frac{1}{\xi_{\mathbf{d}}}\left[\left(-\log\left(1-\frac{1}{T}\right)\right)^{-\xi_{\mathbf{d}}} - 1\right]\right), & \xi_{\mathbf{d}} \neq 0, \\ \sigma_0(\mathbf{d}+\theta)^{-\eta}\left(\mu_t - \log\left(-\log\left(1-\frac{1}{T}\right)\right)\right), & \xi_{\mathbf{d}} = 0. \end{cases} \tag{S4}$$

   After that, we find the the values $I_\Xi(D,T)$ at the discrete durations $\mathbf{d}$ denoted $I_\Xi(D,T)_{\mathbf{d}}$. We obtain the differences in values of $I(\mathbf{d},T)$ and $I_\Xi(D,T)_{\mathbf{d}}$.

$$\Delta_{\mathbf{d}} = I(\mathbf{d},T) - I_\Xi(D,T)_{\mathbf{d}} \tag{S5}$$

   We also obtain the differences in confidence intervals by applying Equation S5 to the quantiles of $I(\mathbf{d},T)$ and $I_\Xi(D,T)_{\mathbf{d}}$. Then we linearly interpolate $\Delta_{\mathbf{d}}$ using the same procedure as outlined in Equation S1 and S2.

$$\Delta_{\mathbf{d},i,k} = \Delta_{\mathbf{d},i} + k \times \frac{\Delta_{\mathbf{d},i+1} + \Delta_{\mathbf{d},i}}{m} \quad \text{for } k = 1, 2, \ldots, m \tag{S6}$$

   Finally, the adjusted IDF curve $\hat{I}_{i,k}$ and confidence intervals can be obtained by adding $\Delta_{\mathbf{d},i,k}$ to $I_\Xi(D,T)$:

$$\hat{I}_{i,k} = I_\Xi(D,T) + \Delta_{\mathbf{d},i,k} \tag{S7}$$

   This approach ensures that the final IDF curve accurately reflects the actual return levels at the specified discrete durations.

1.2 Widely applicable information criterion

The Widely Applicable Information Criterion (WAIC) is a statistical measure used for model comparison and selection. It is aimed to achieve parsimony by balancing model fit and complexity, which provides a measure of a model's predictive validity. The WAIC is similar to the Akaike information criterion (AIC) and Bayesian information criterion (BIC), which helps choose the best statistical model for the data. However, these methods only rely on point estimates, whereas the WAIC uses the entire posterior distribution of parameters providing a more extensive picture of the model. This is particularly useful for Bayesian models. The formula for the WAIC is:

$$WAIC = -2\,(LPPD(y|\theta) - d_e \tag{S8}$$

$LPPD$ is the log posterior predictive density of each data point y. Estimating $LPPD$ for data point y, the likelihood of each data point is obtained for the entire posterior parameter sample. The model complexity, d_e, is defined as the sum of the variances of the log-likelihood, calculated for each individual data point, where these variances are evaluated over the posterior distribution of the model parameters. A higher de indicates a more complex model. We calculate the WAIC for the four Bayesian EVT models only.

2 Supplementary Figures

2.1 Return level plots

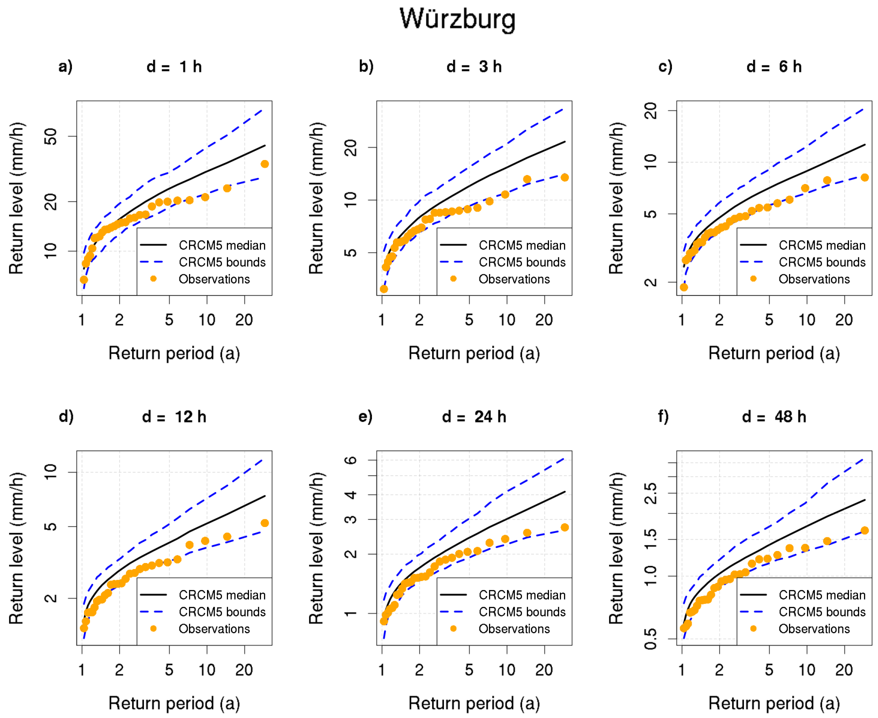

Figure S1: Return level for the locality of Würzburg

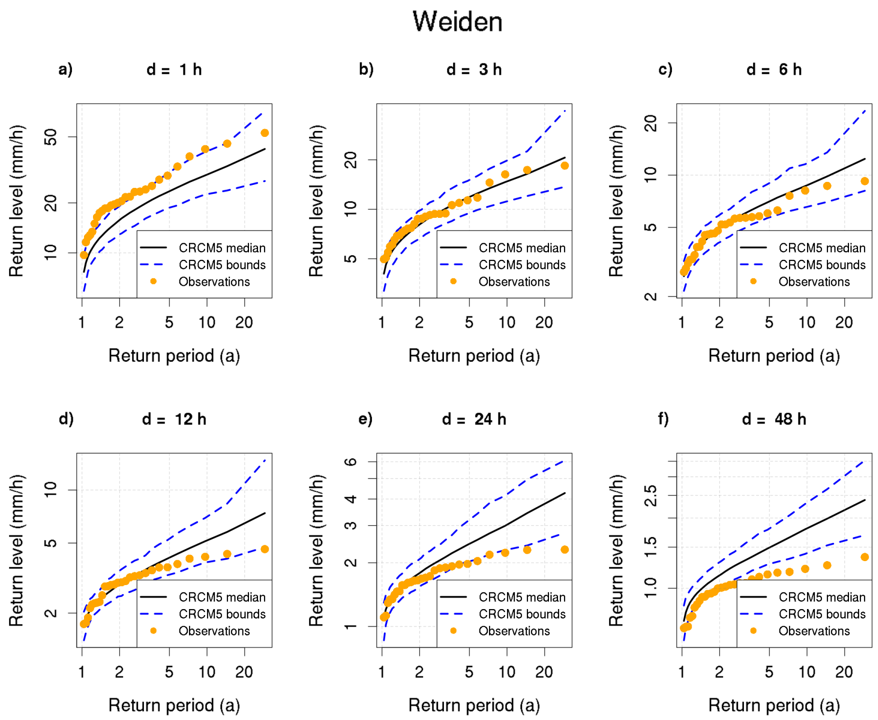

Figure S2: Return level for the locality of Weiden

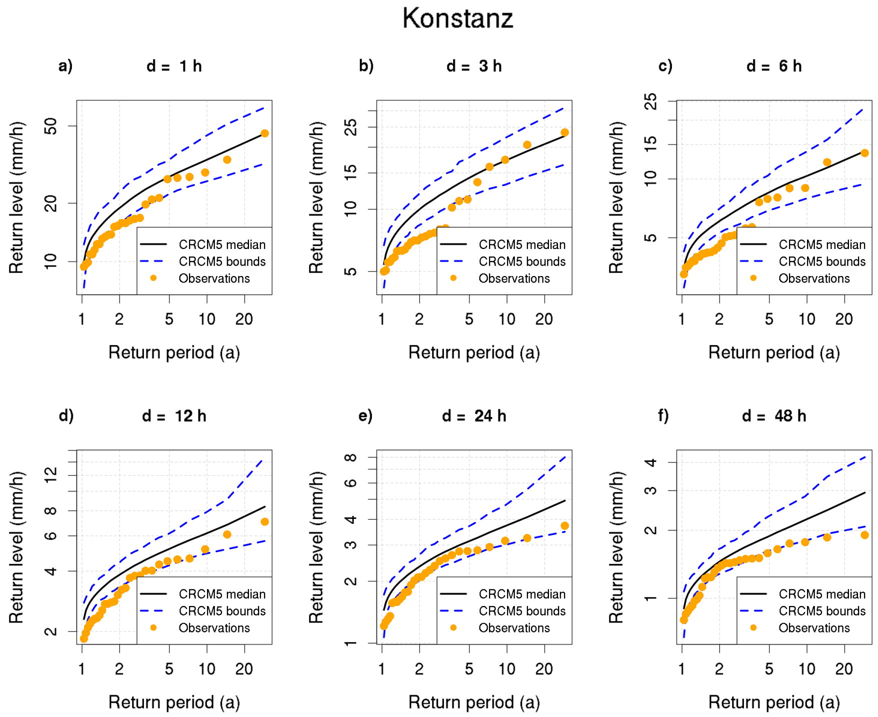

Figure S3: Return level for the locality of Konstanz

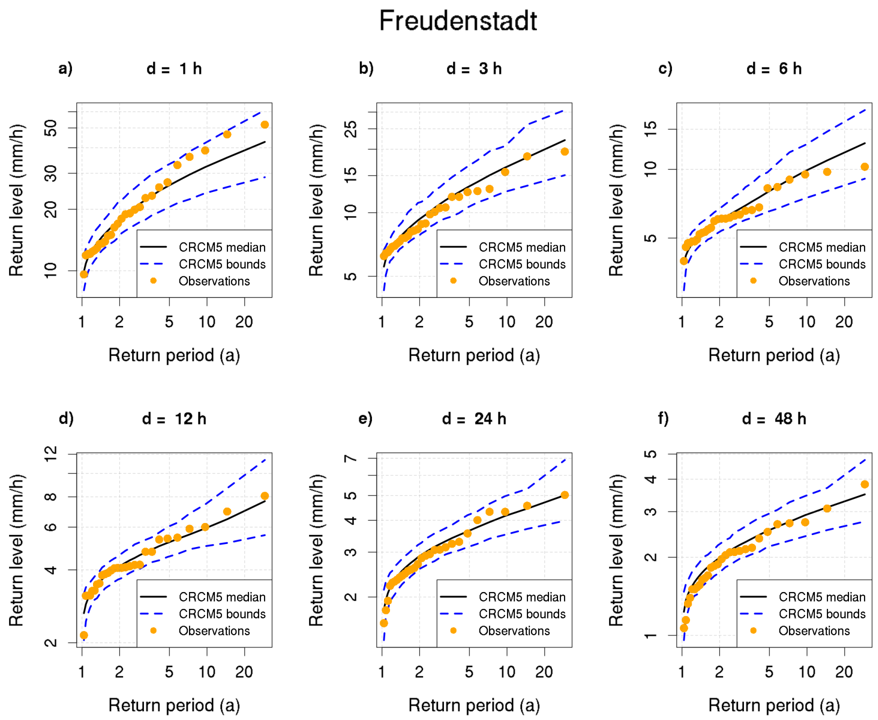

Figure S4: Return level for the locality of Freudenstadt

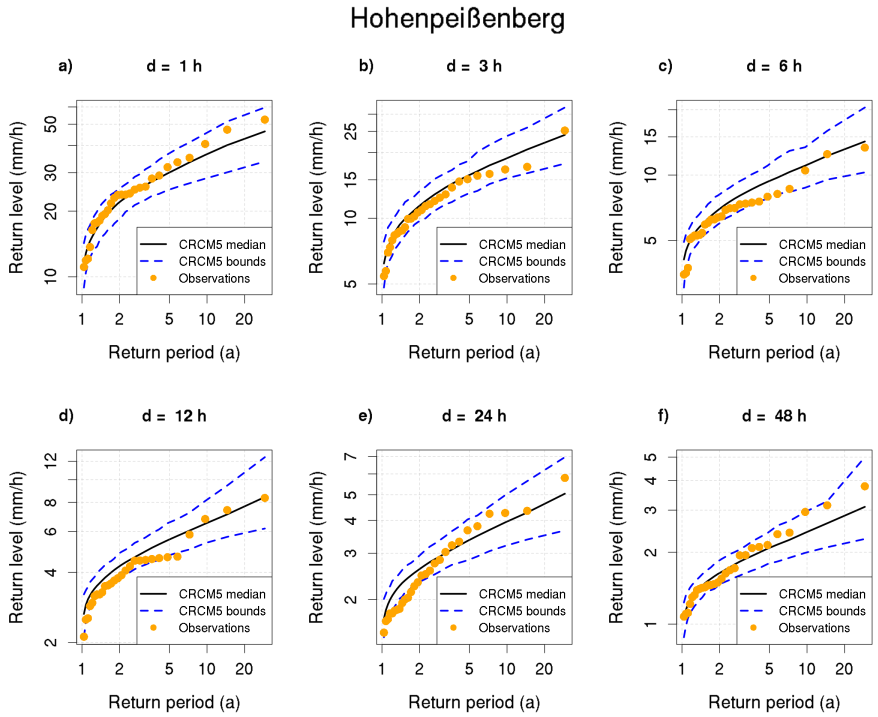

Figure S5: Return level for the locality of Hohenpeißenberg

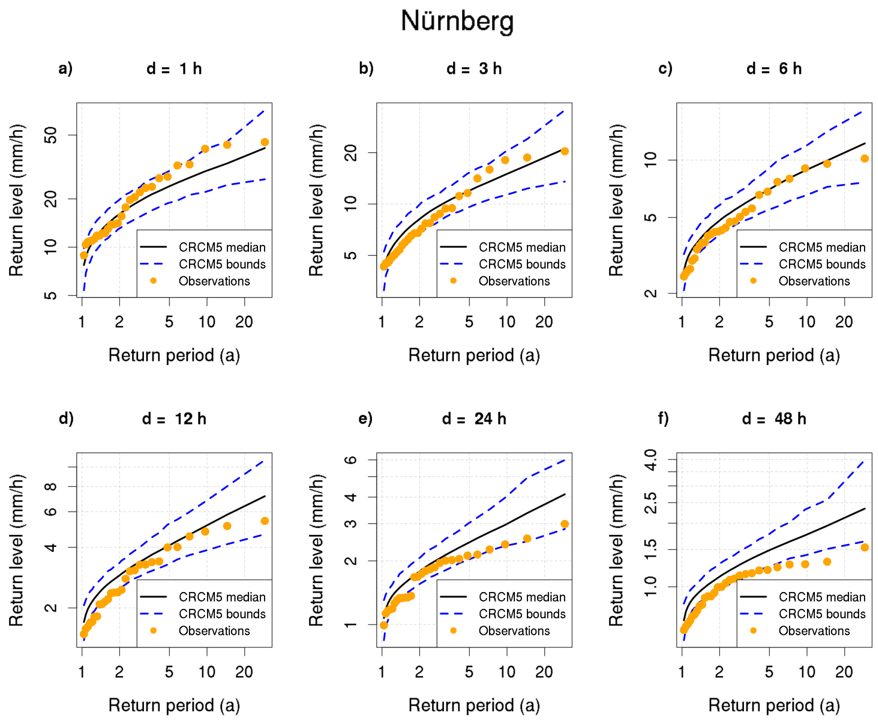

Figure S6: Return level for the locality of Nürnberg

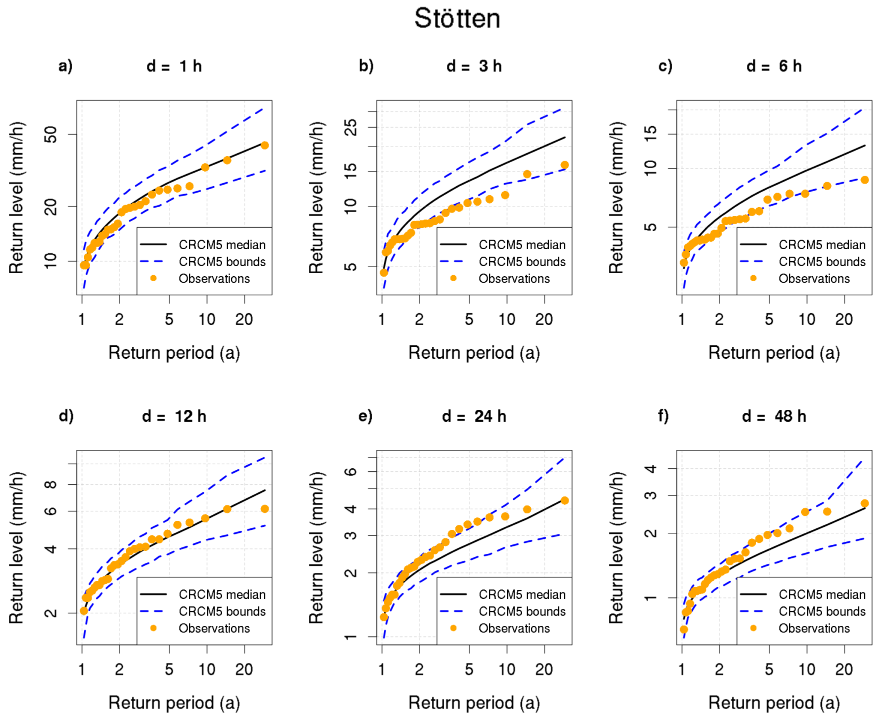

Figure S7: Return level for the locality of Stötten

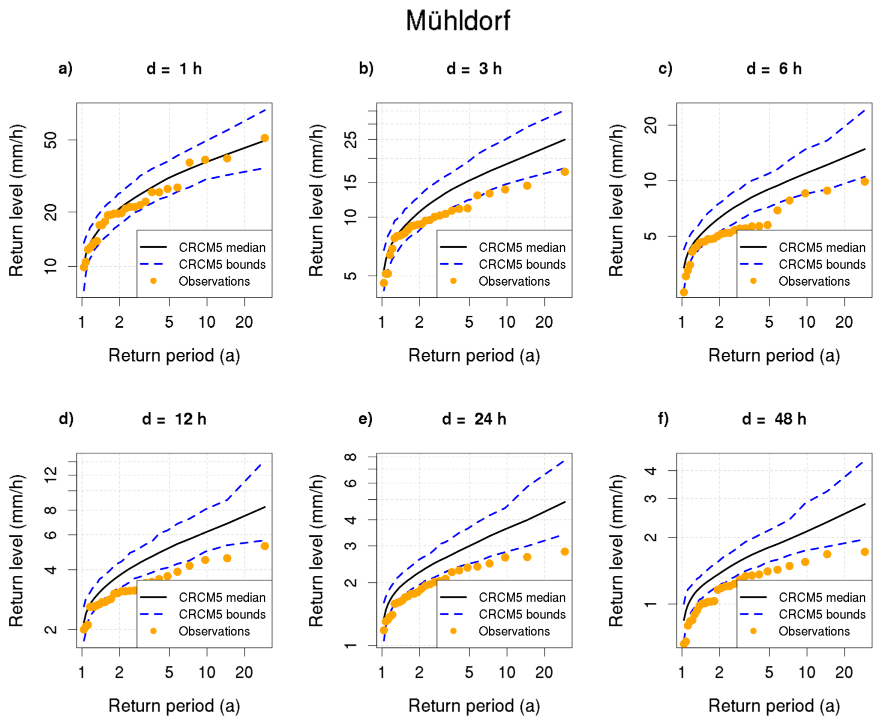

Figure S8: Return level for the locality of Mühldorf

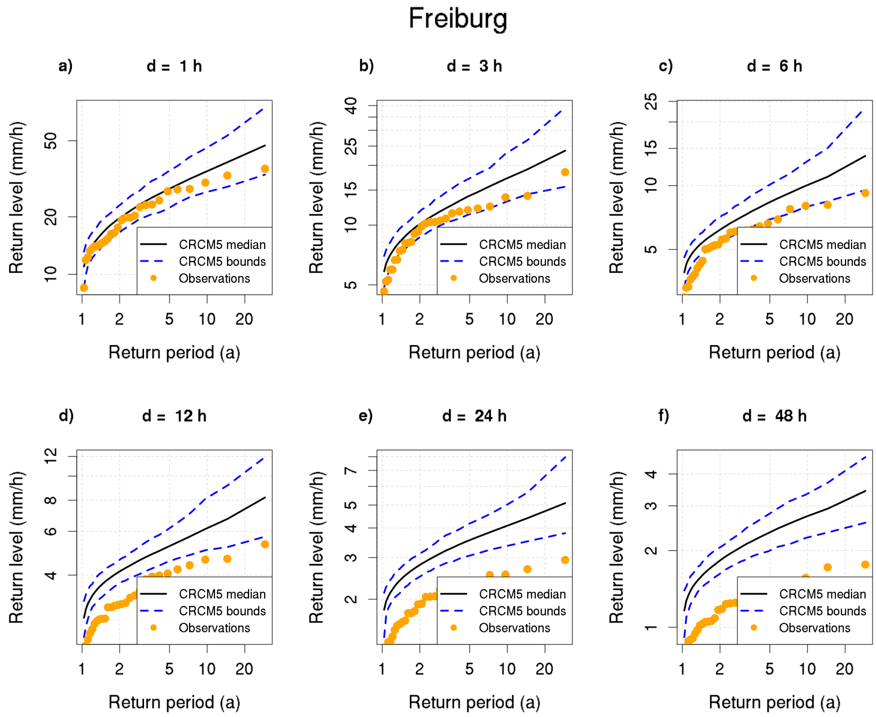

Figure S9: Return level for the locality of Freiburg

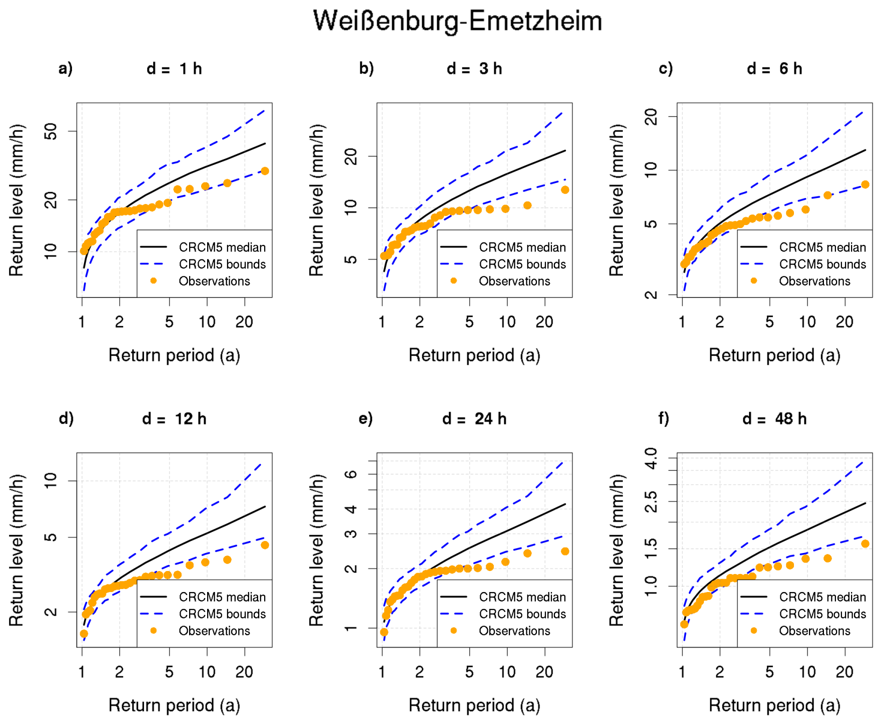

Figure S10: Return level for the locality of Weißenburg-Emetzheim

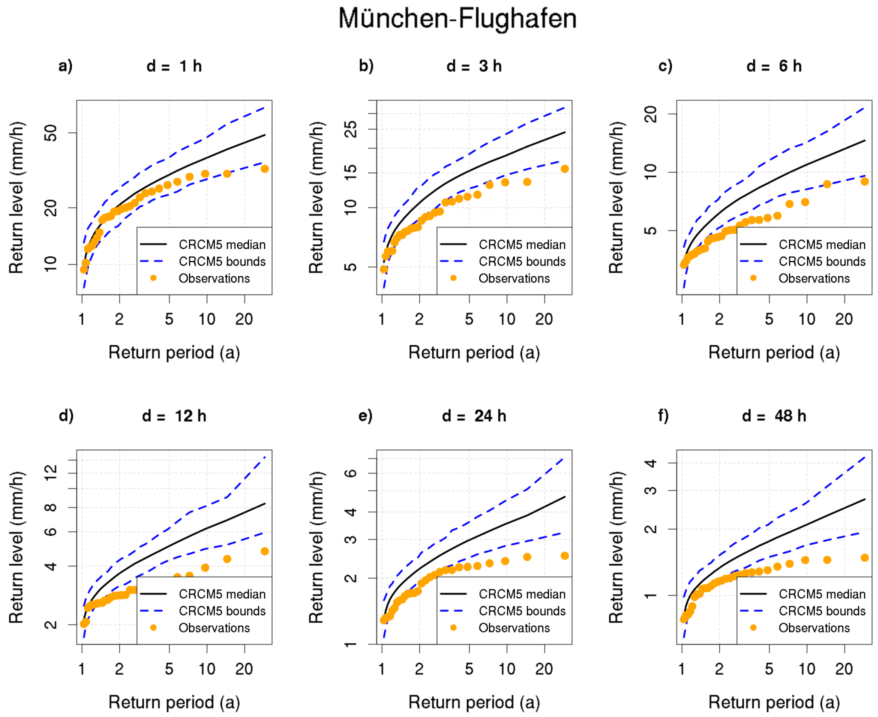

Figure S11: Return level for the locality of München-Flughafen

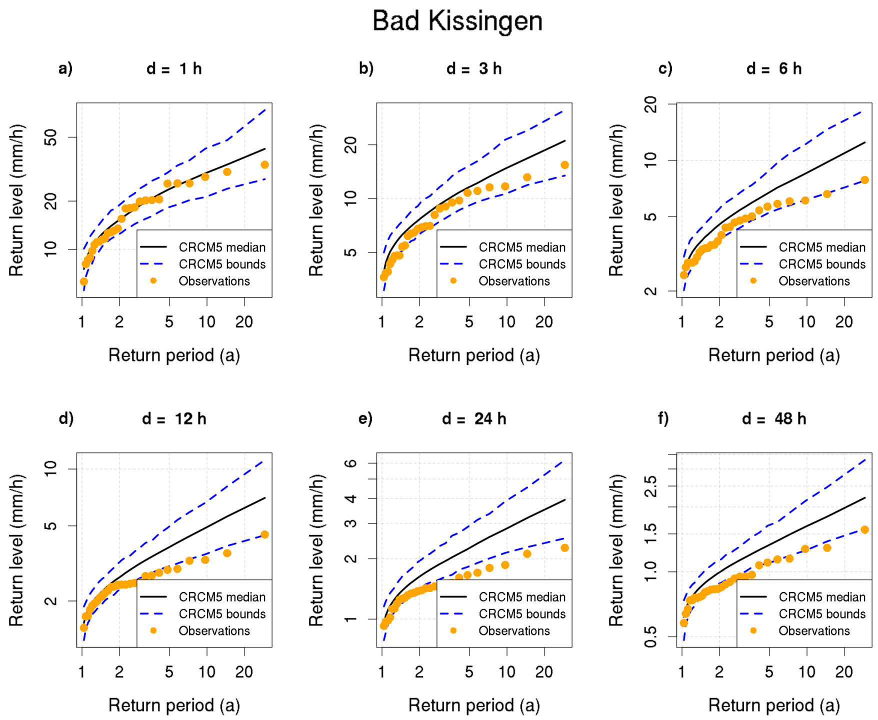

Figure S12: Return level for the locality of Bad Kissingen

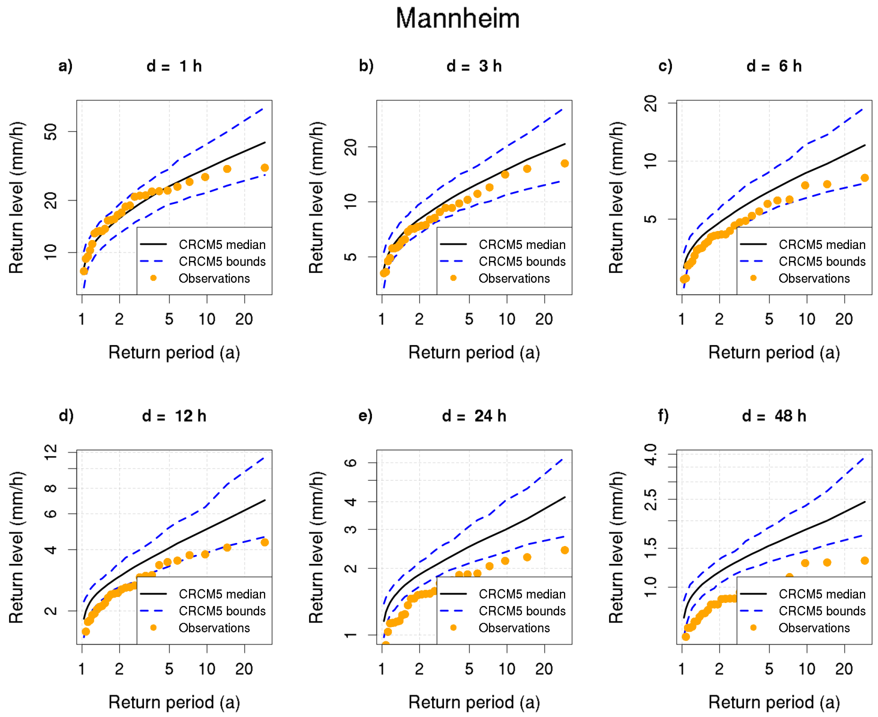

Figure S13: Return level for the locality of Mannheim

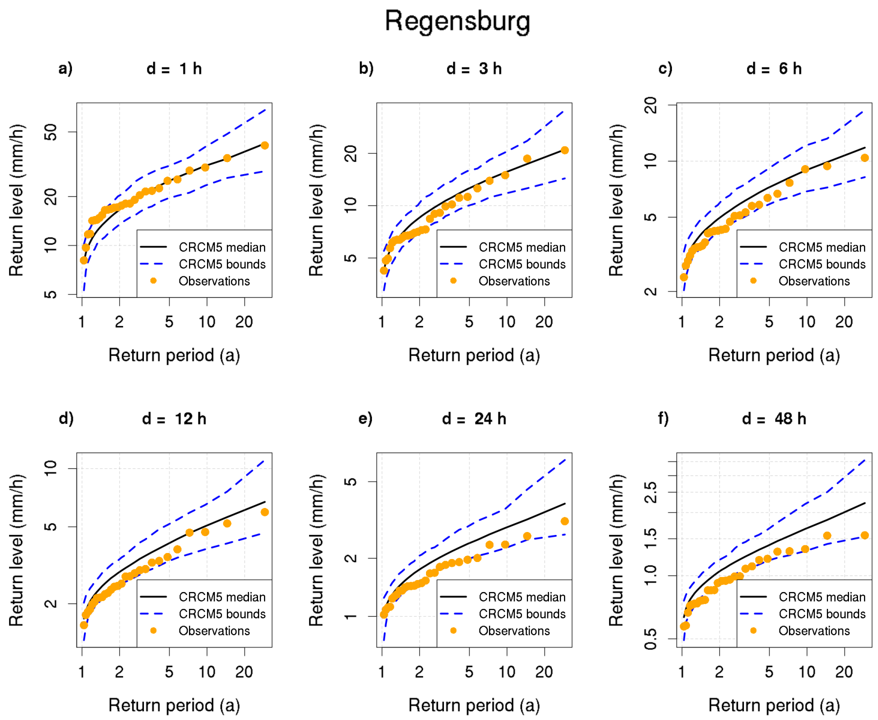

Figure S14: Return level for the locality of Regensburg

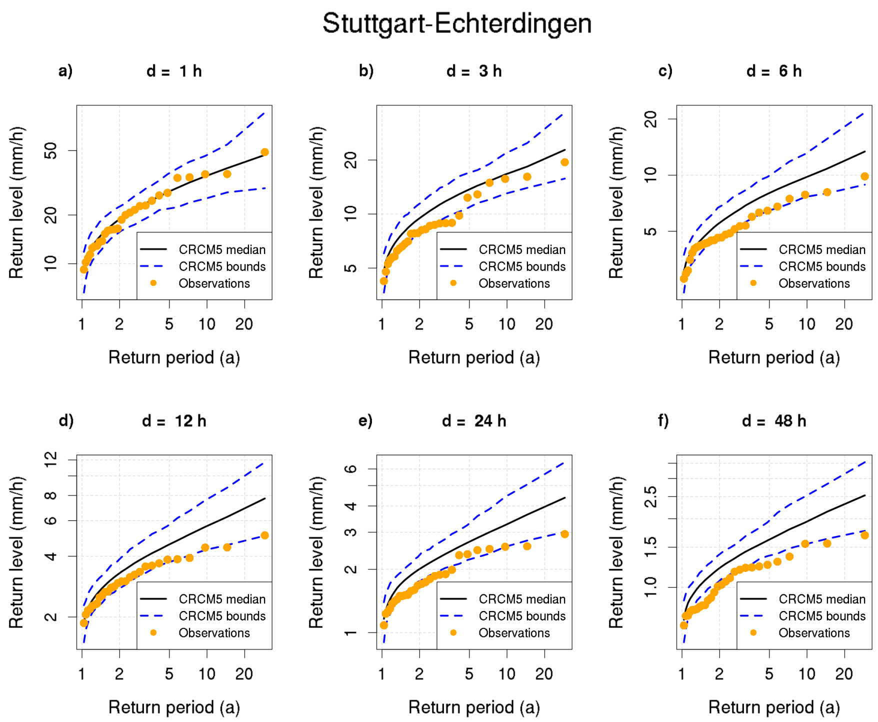

Figure S15: Return level for the locality of Stuttgart-Echterdingen

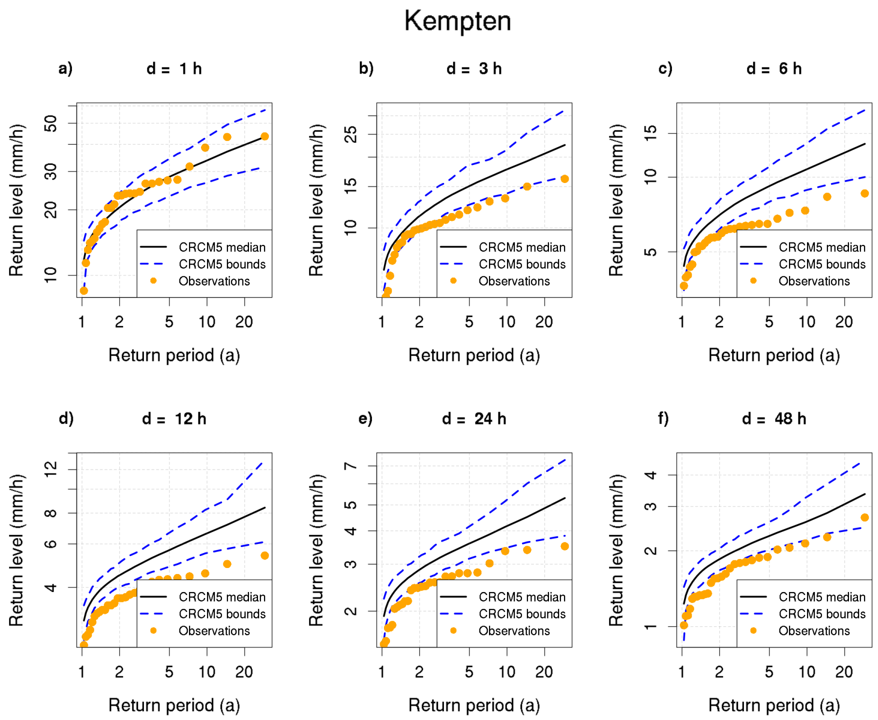

Figure S16: Return level for the locality of Kempten

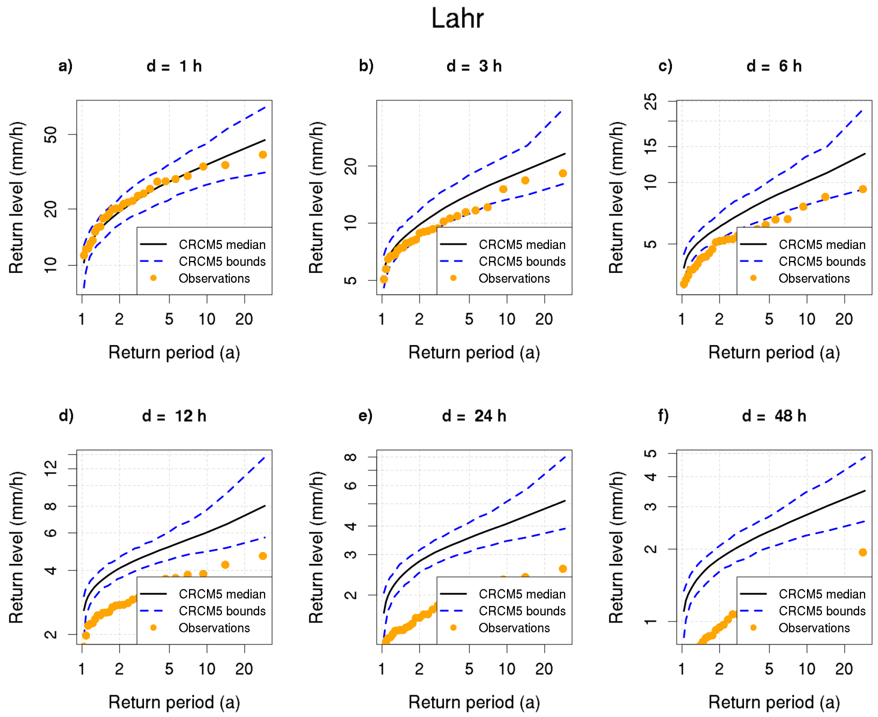

Figure S17: Return level for the locality of Lahr

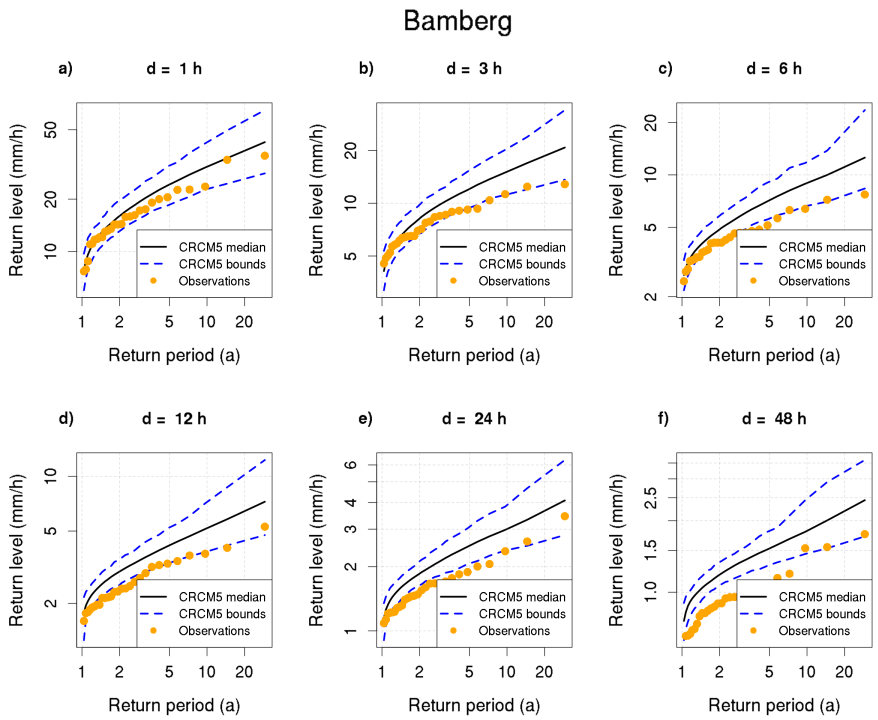

Figure S18: Return level for the locality of Bamberg

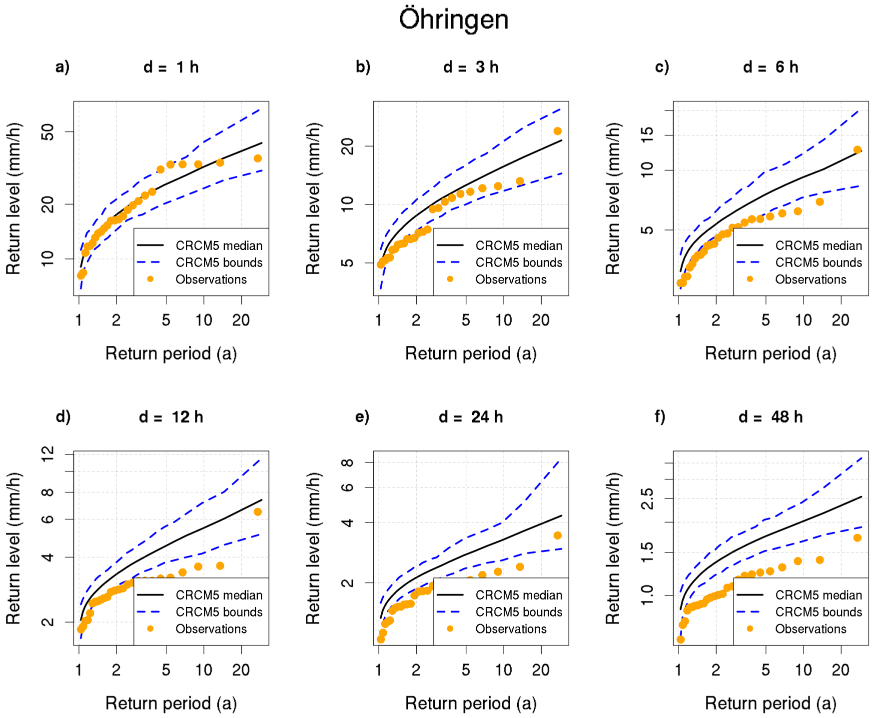

Figure S19: Return level for the locality of Öhringen

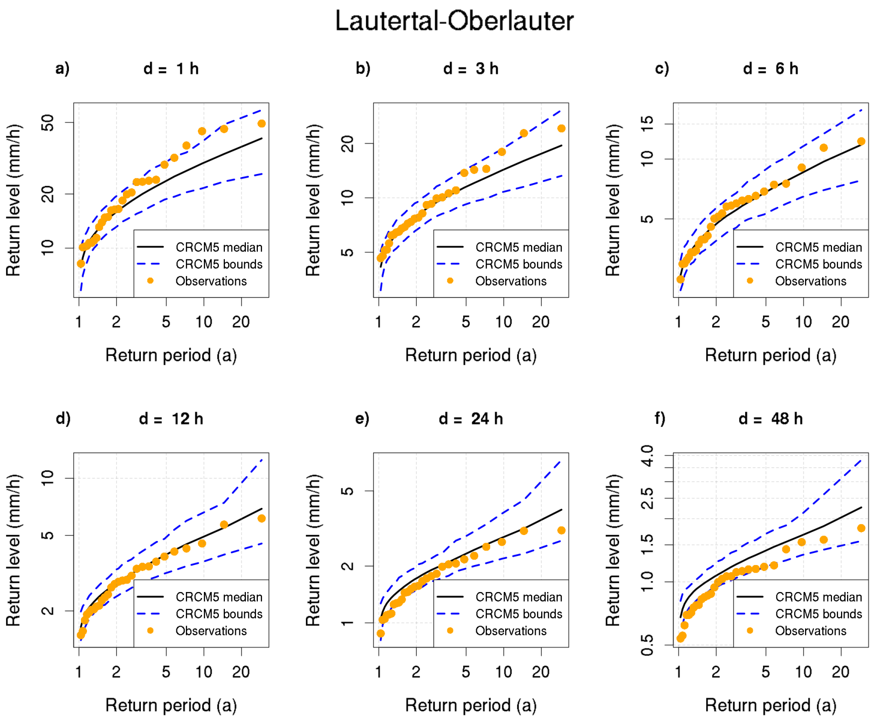

Figure S20: Return level for the locality of Lautertal-Oberlauter

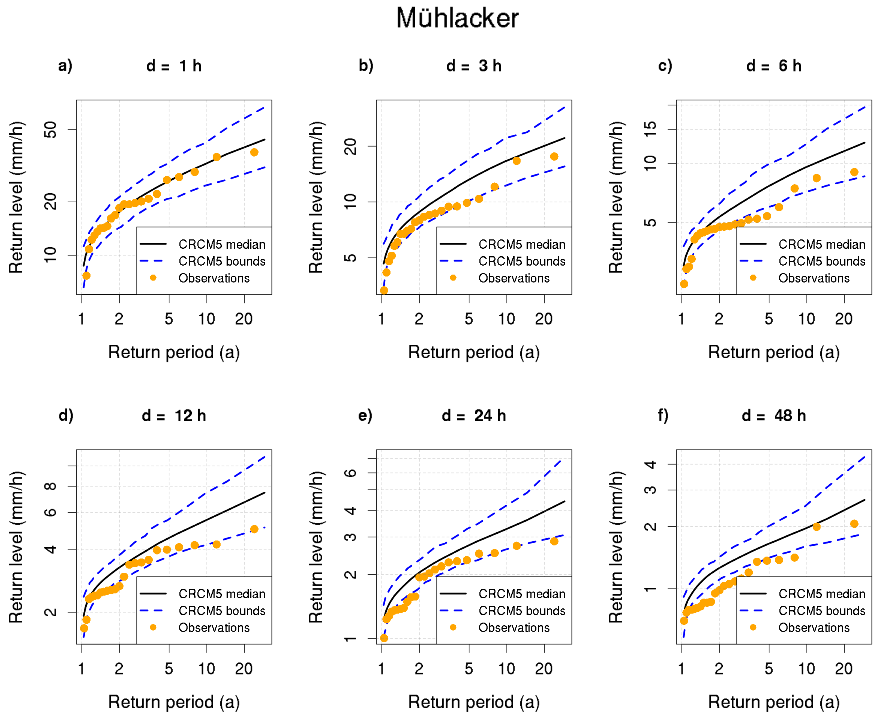

Figure S21: Return level for the locality of Mühlacker

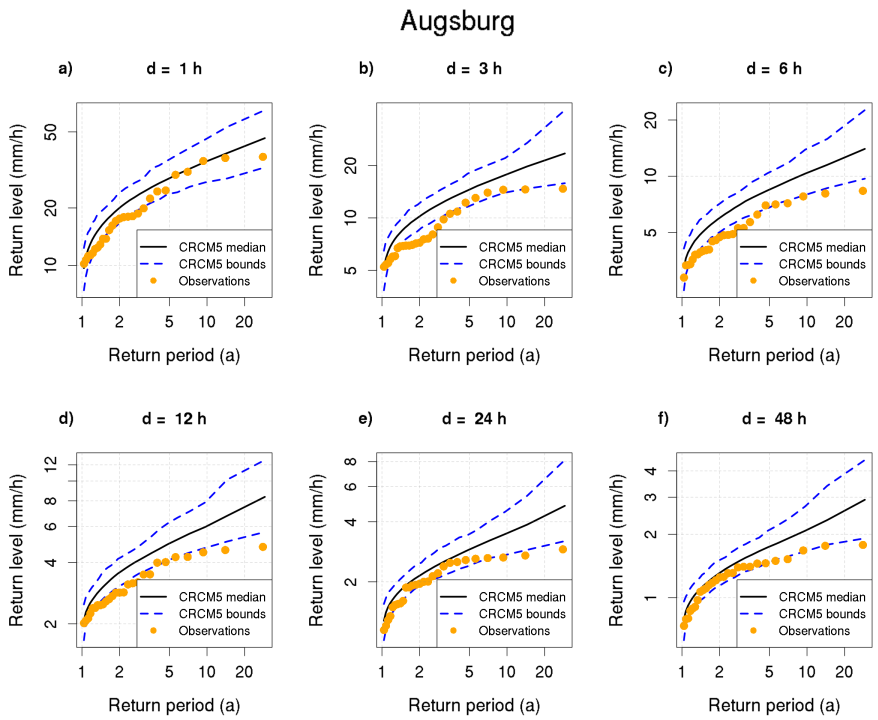

Figure S22: Return level for the locality of Augsburg

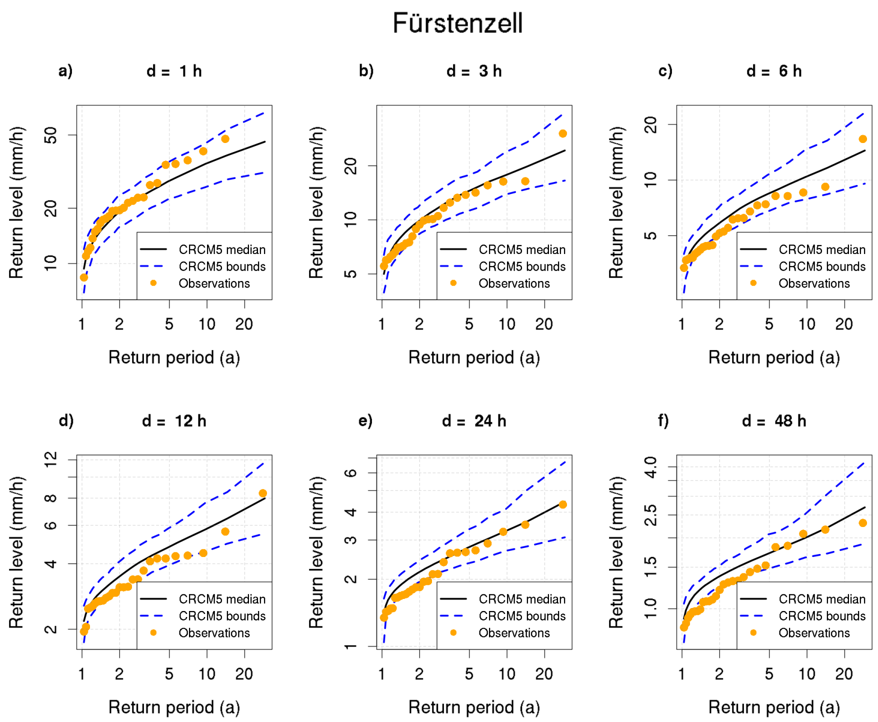

Figure S23: Return level for the locality of Fürstenzell

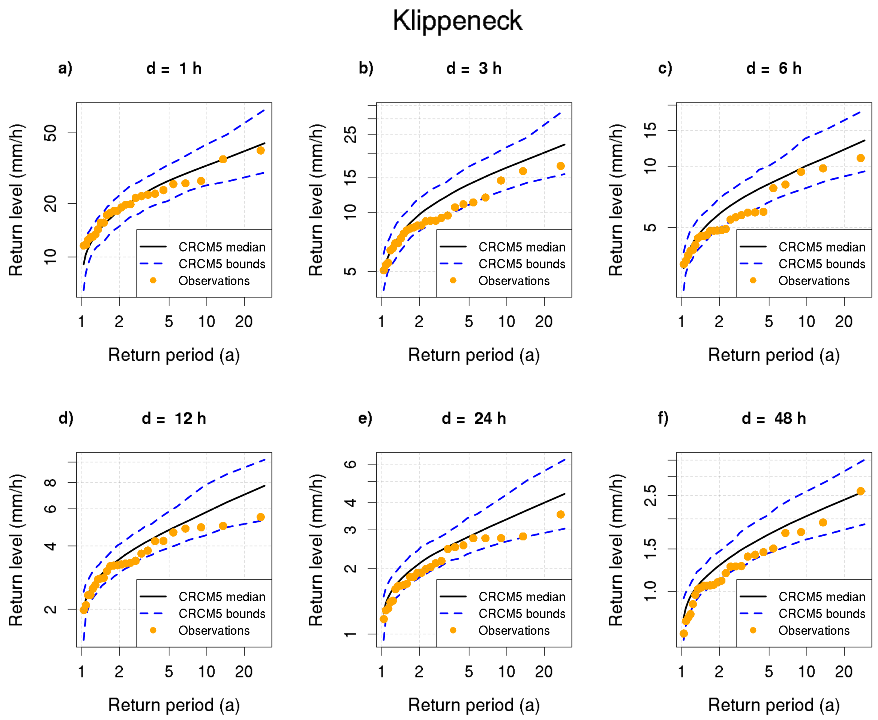

Figure S24: Return level for the locality of Klippeneck

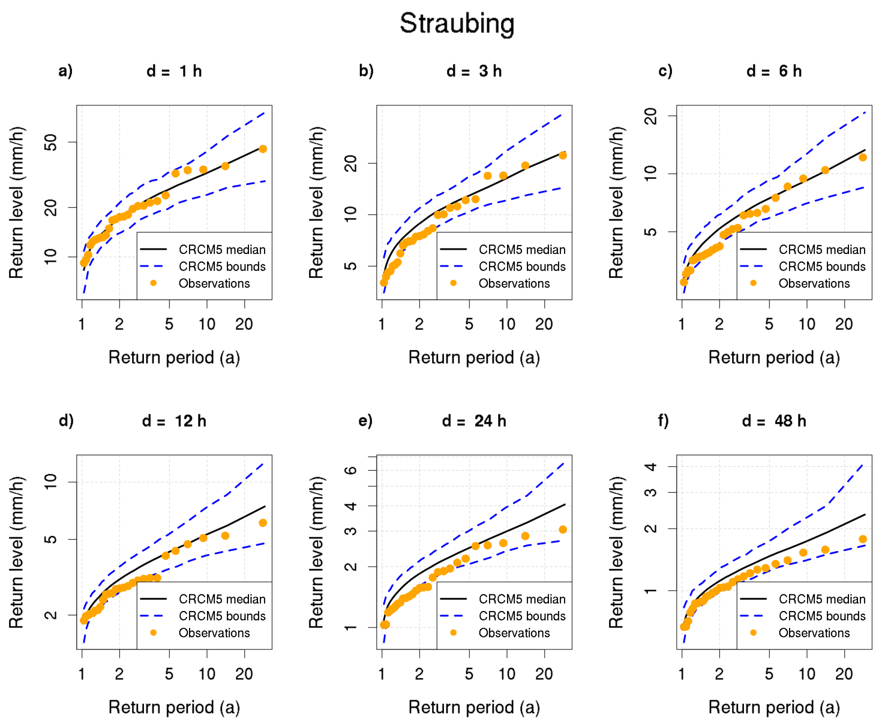

Figure S25: Return level for the locality of Straubing

2.2 IDF curve plots

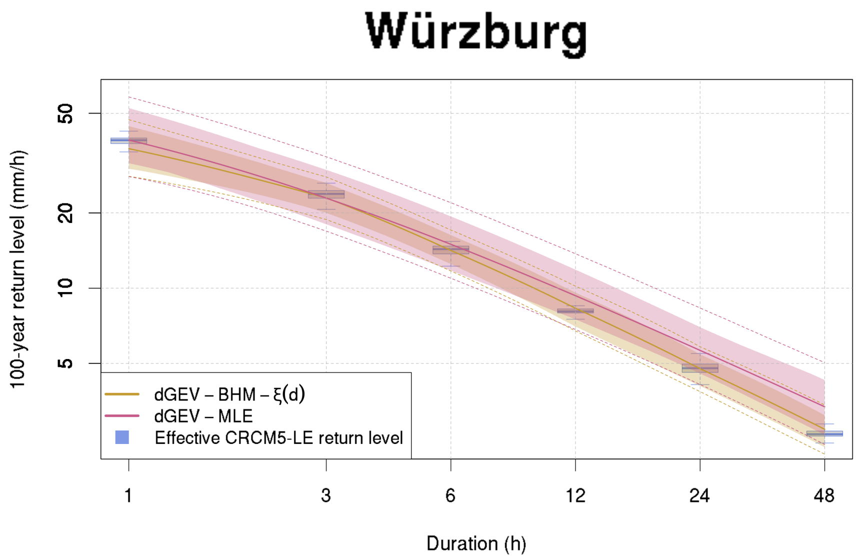

Figure S26: IDF curve for the locality of Würzburg

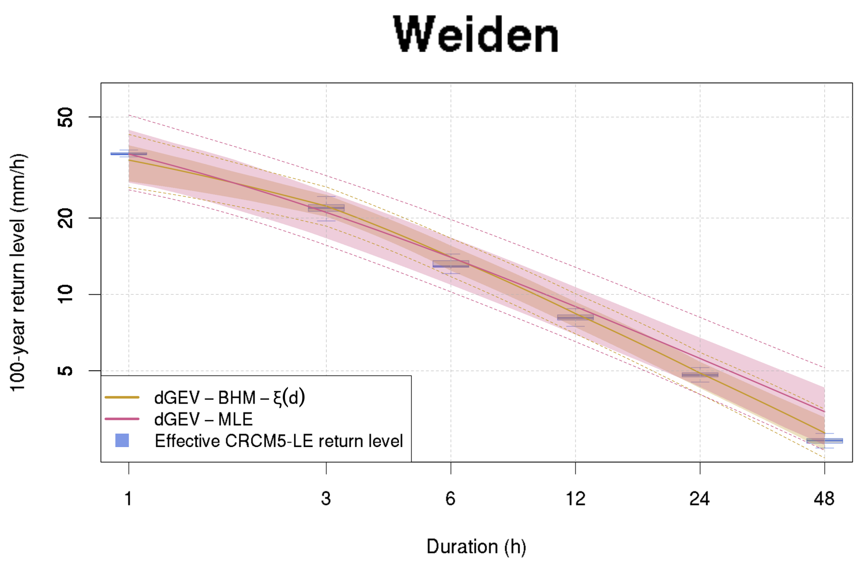

Figure S27: IDF curve for the locality of Weiden

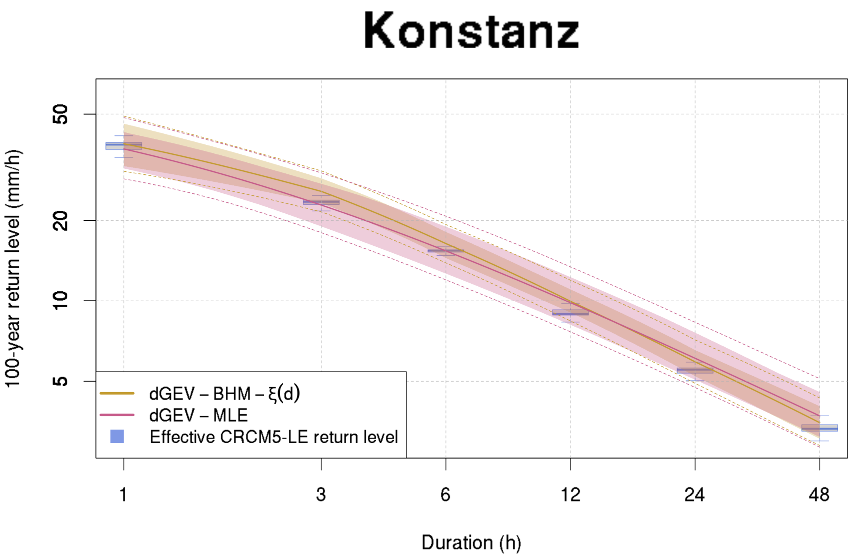

Figure S28: IDF curve for the locality of Konstanz

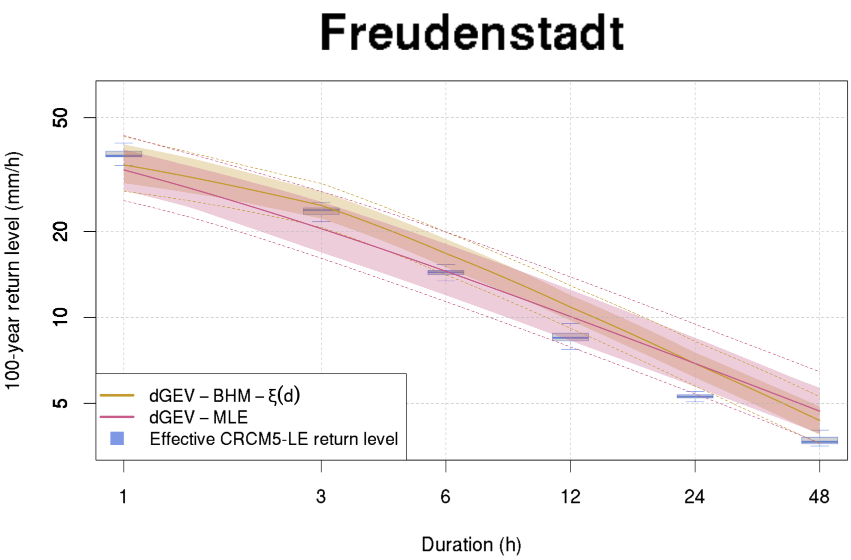

Figure S29: IDF curve for the locality of Freudenstadt

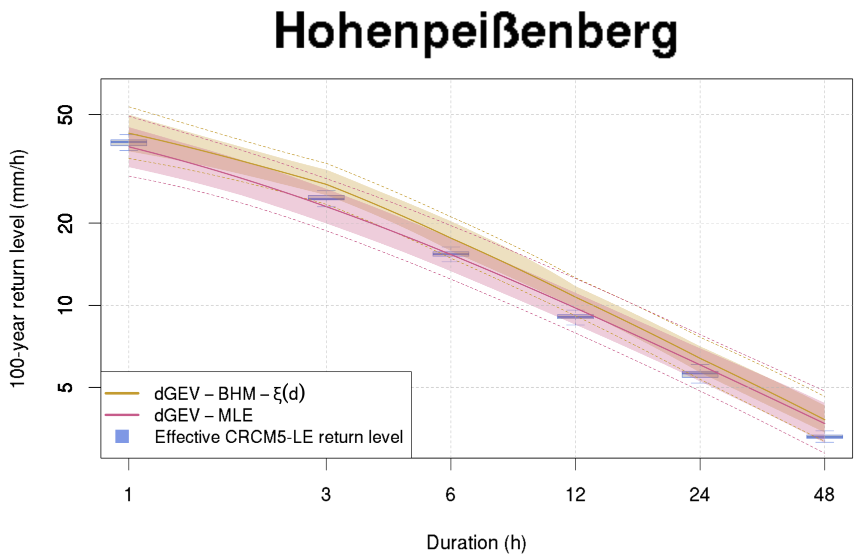

Figure S30: IDF curve for the locality of Hohenpeißenberg

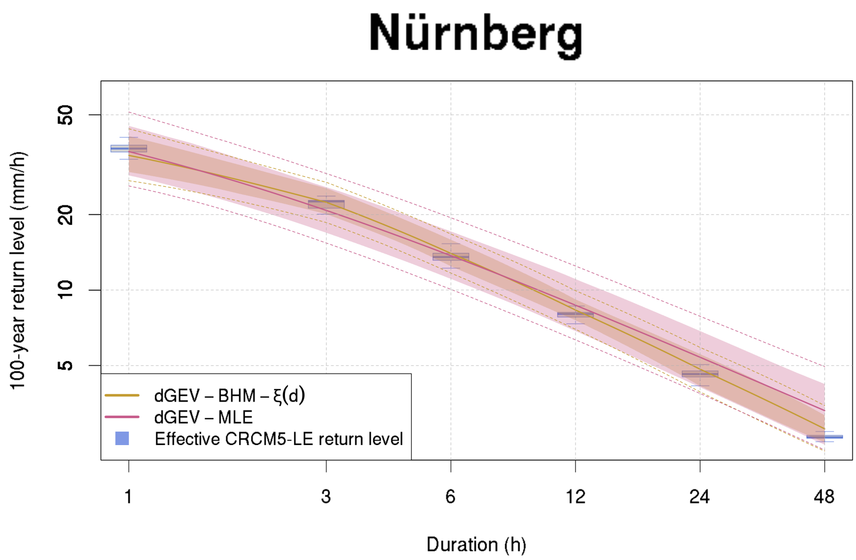

Figure S31: IDF curve for the locality of Nürnberg

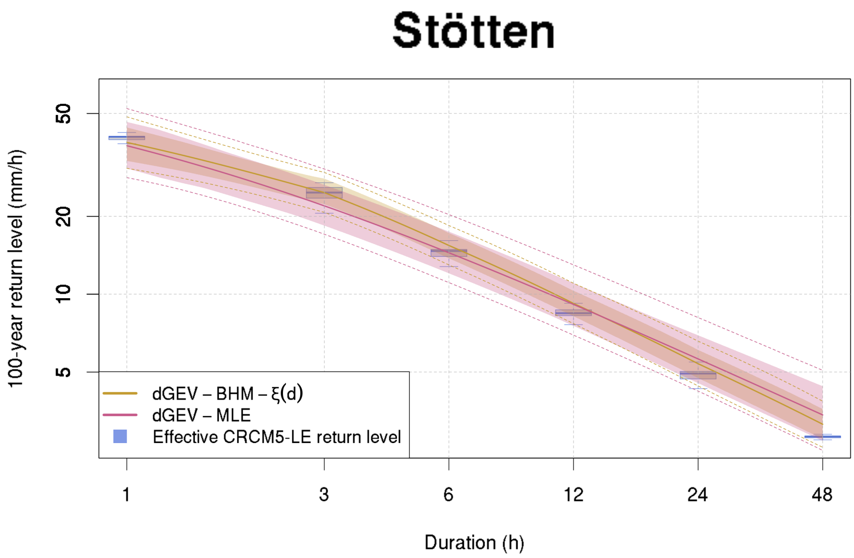

Figure S32: IDF curve for the locality of Stötten

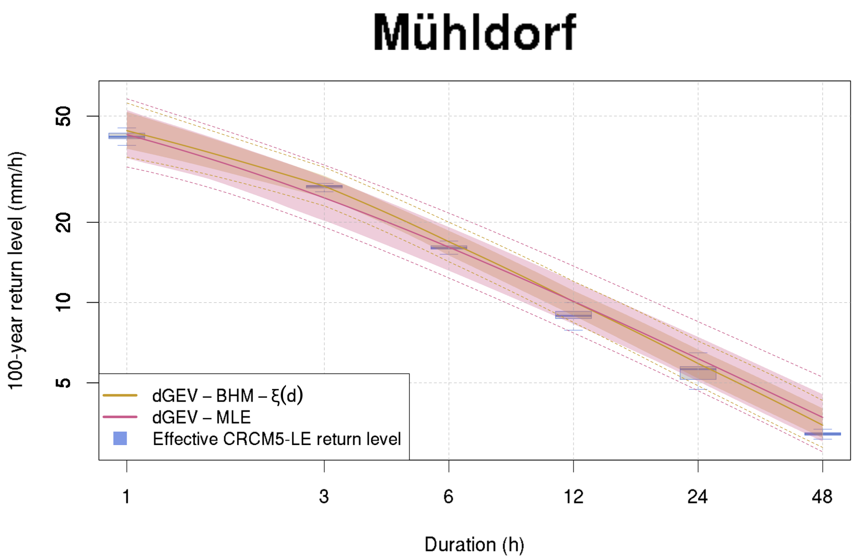

Figure S33: IDF curve for the locality of Mühldorf

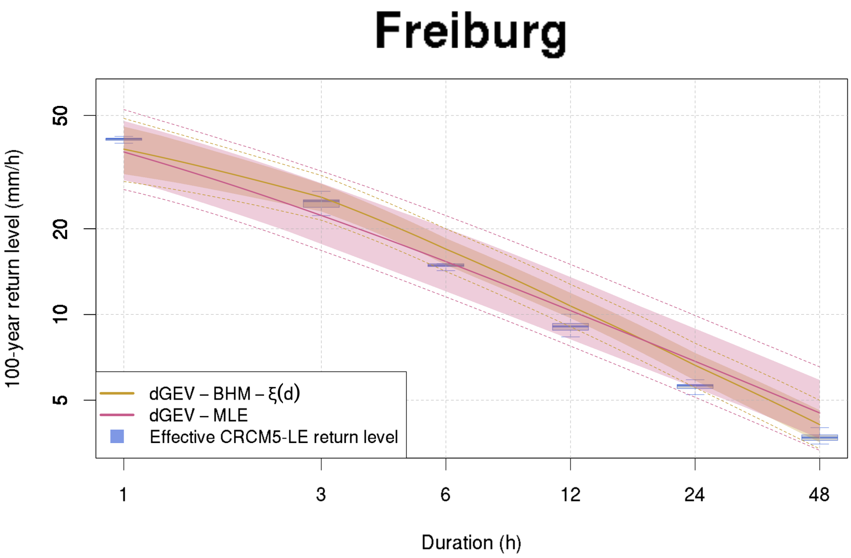

Figure S34: IDF curve for the locality of Freiburg

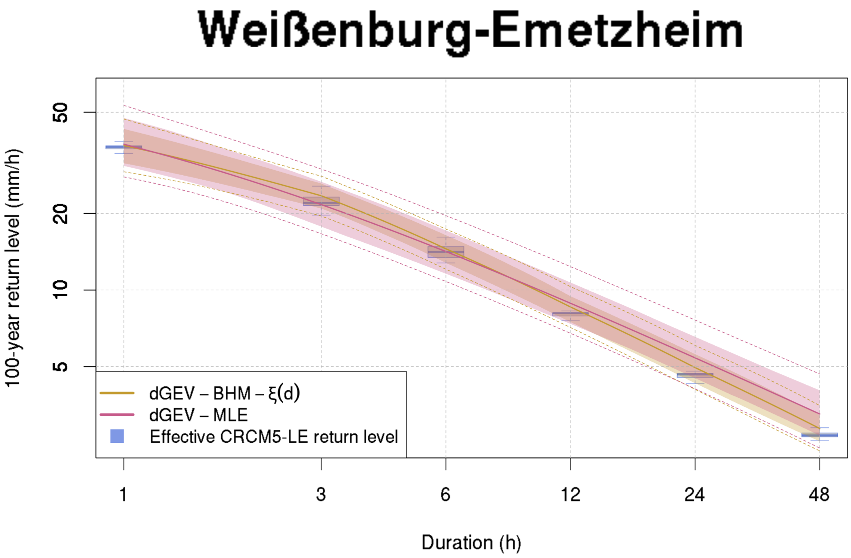

Figure S35: IDF curve for the locality of Weißenburg-Emetzheim

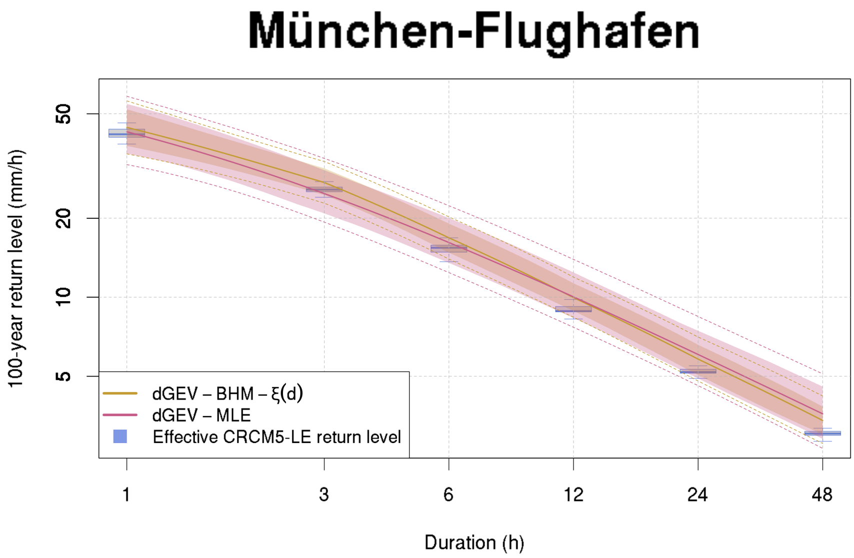

Figure S36: IDF curve for the locality of München-Flughafen

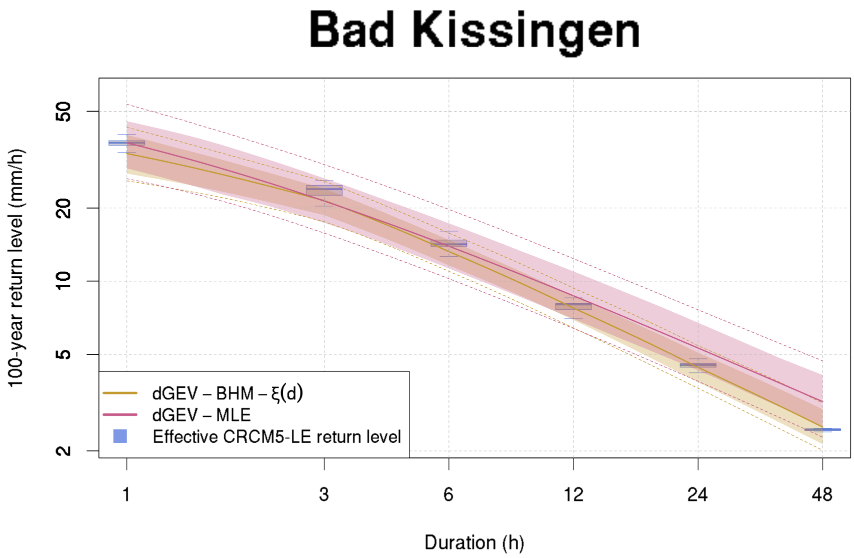

Figure S37: IDF curve for the locality of Bad Kissingen

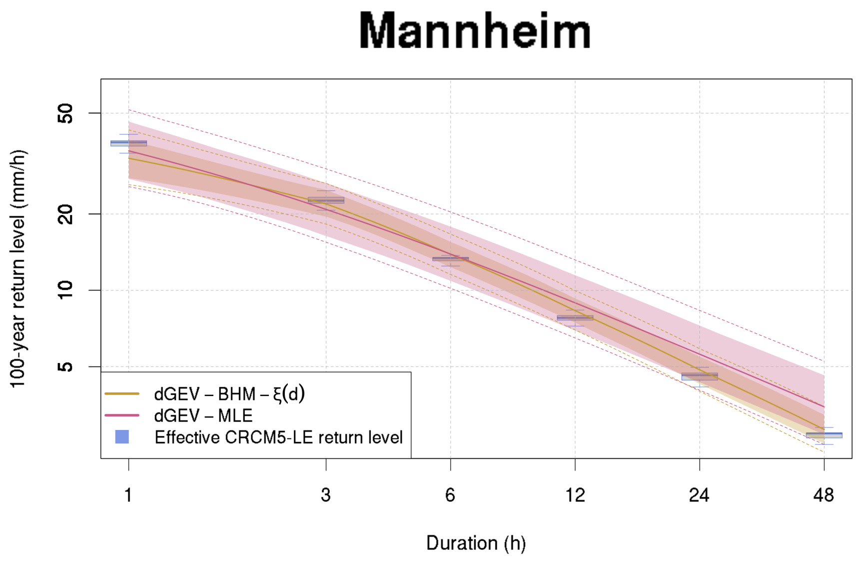

Figure S38: IDF curve for the locality of Mannheim

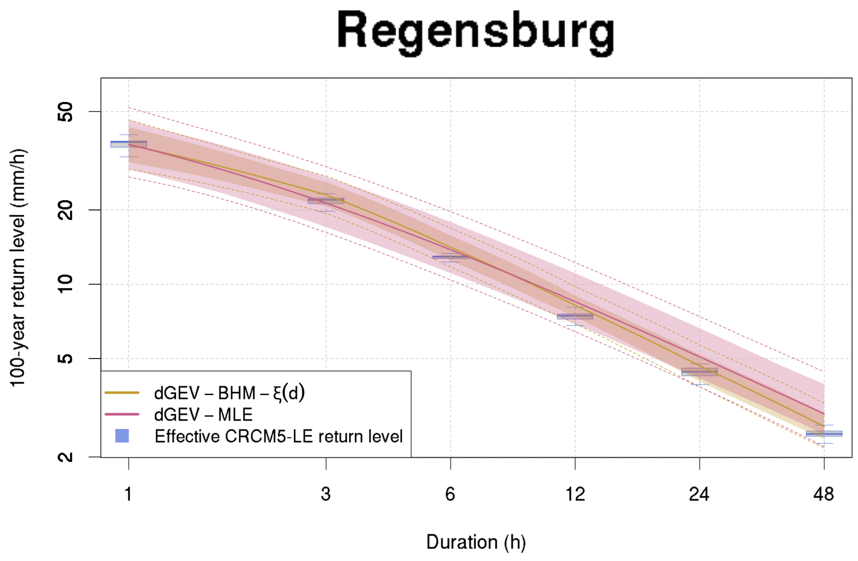

Figure S39: IDF curve for the locality of Regensburg

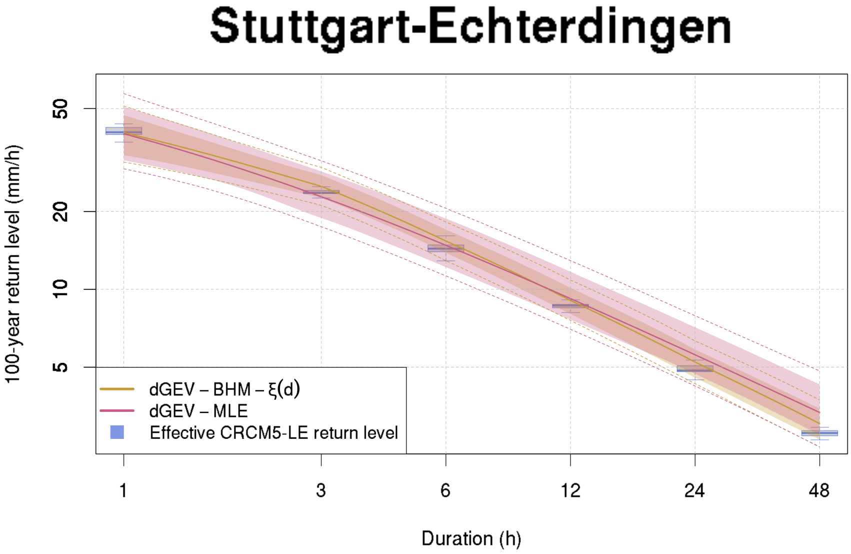

Figure S40: IDF curve for the locality of Stuttgart-Echterdingen

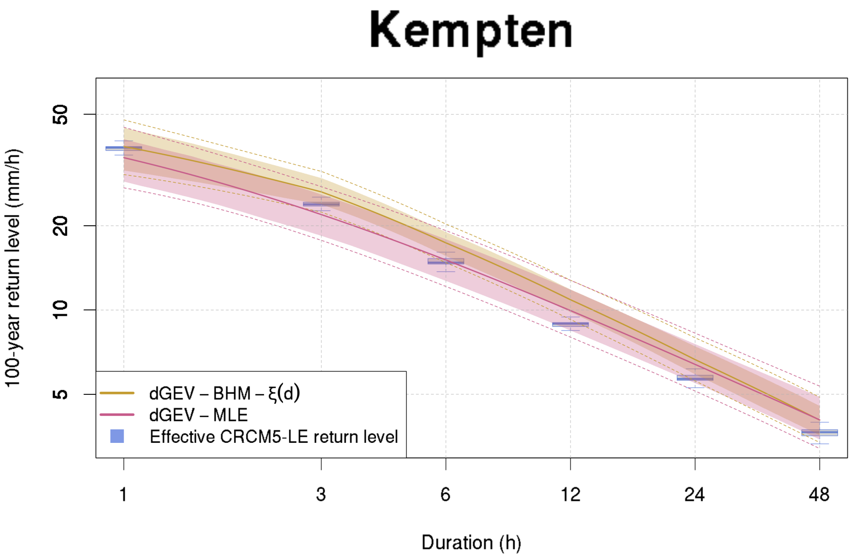

Figure S41: IDF curve for the locality of Kempten

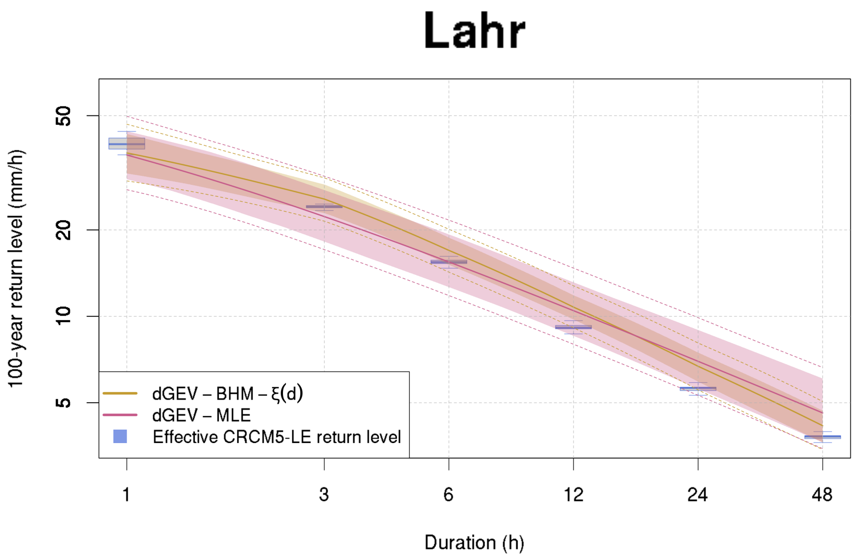

Figure S42: IDF curve for the locality of Lahr

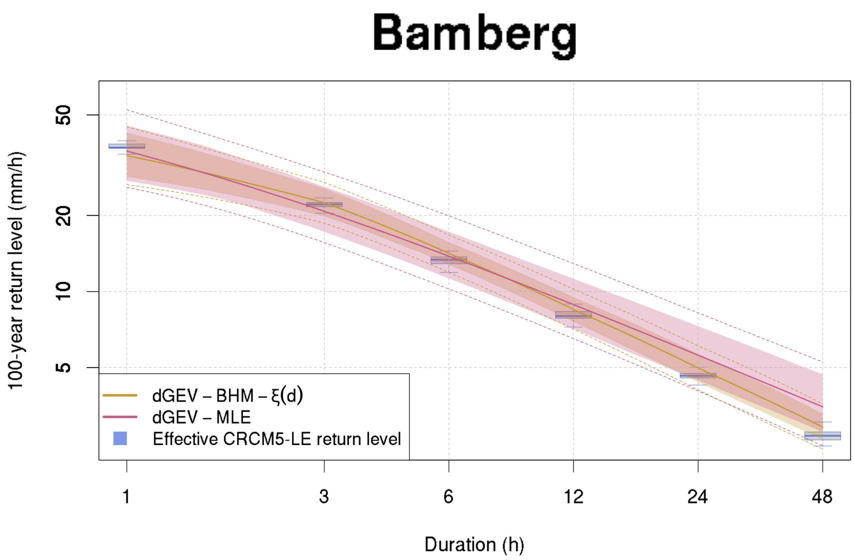

Figure S43: IDF curve for the locality of Bamberg

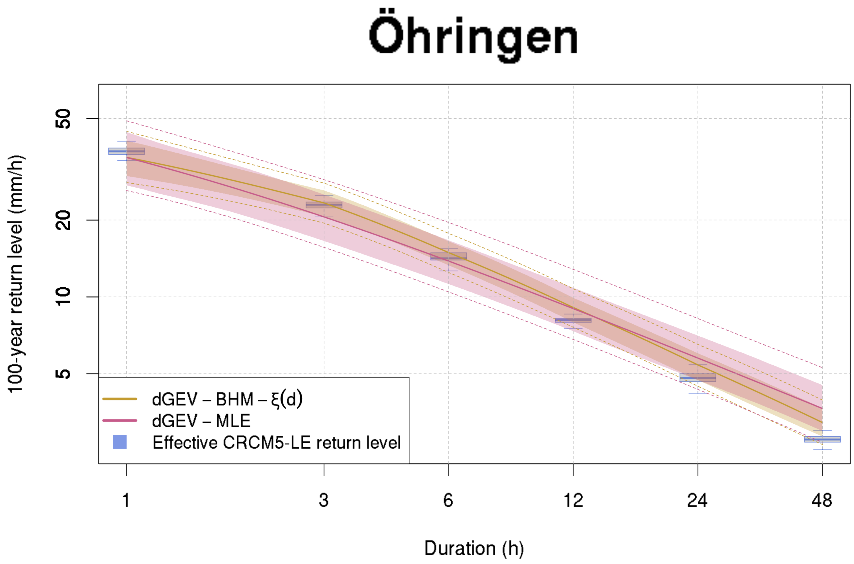

Figure S44: IDF curve for the locality of Öhringen

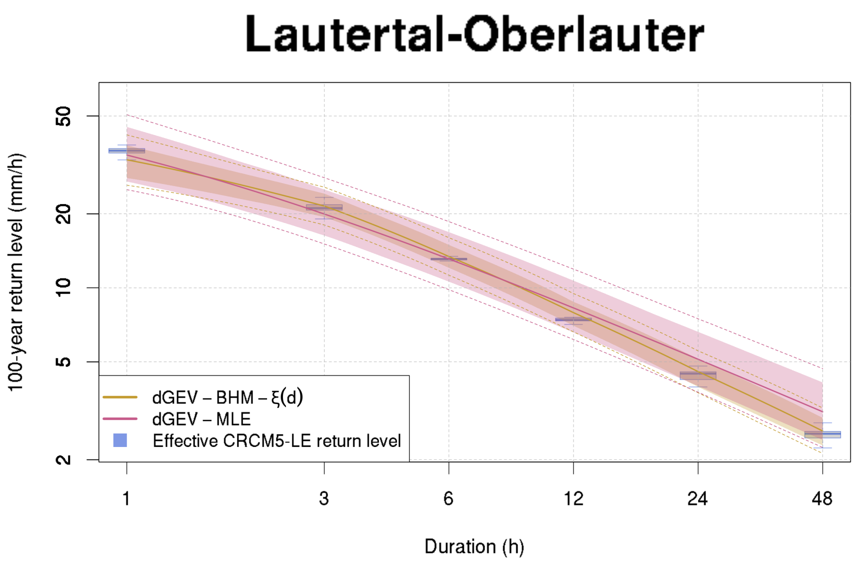

Figure S45: IDF curve for the locality of Lautertal-Oberlauter

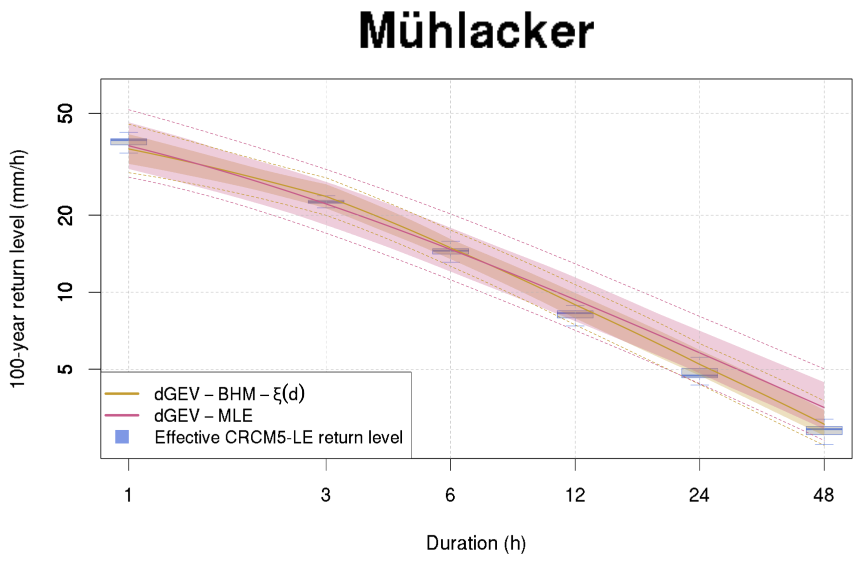

Figure S46: IDF curve for the locality of Mühlacker

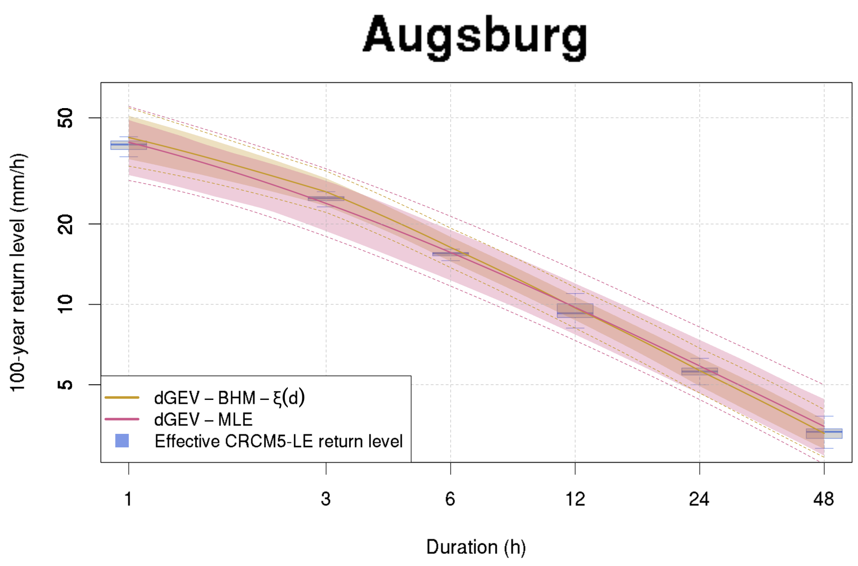

Figure S47: IDF curve for the locality of Augsburg

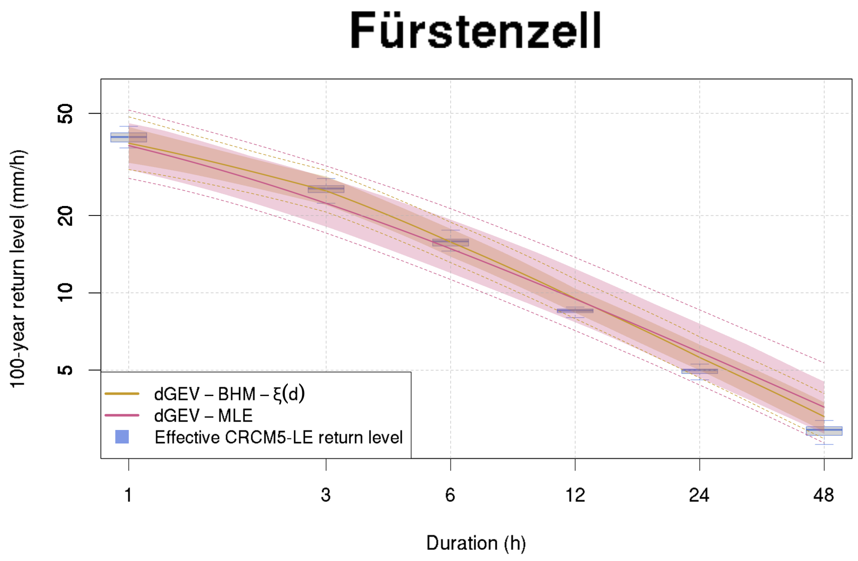

Figure S48: IDF curve for the locality of Fürstenzell

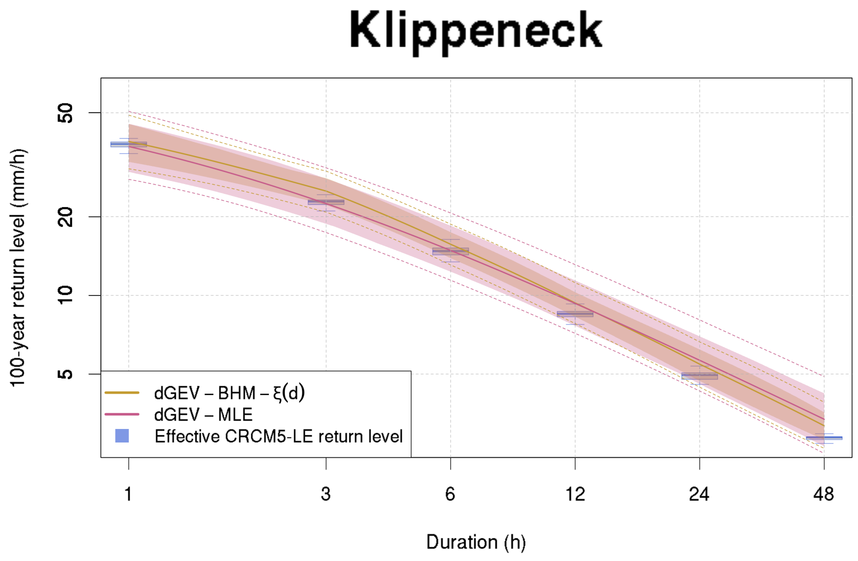

Figure S49: IDF curve for the locality of Klippeneck

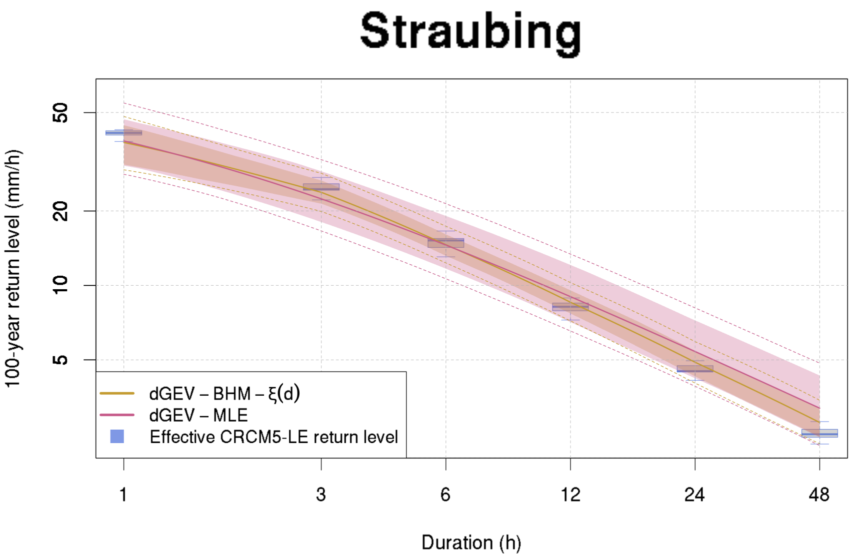

Figure S50: IDF curve for the locality of Straubing

2.3 Simple scaling approach

In this figure we illustrate the ratio between μ and σ. We fit the full 2000-year CRCM5-LE to showing a clear linear relationship between the fitted parameters, see Figure S51 below. This justifies the use of our dGEV formulation.

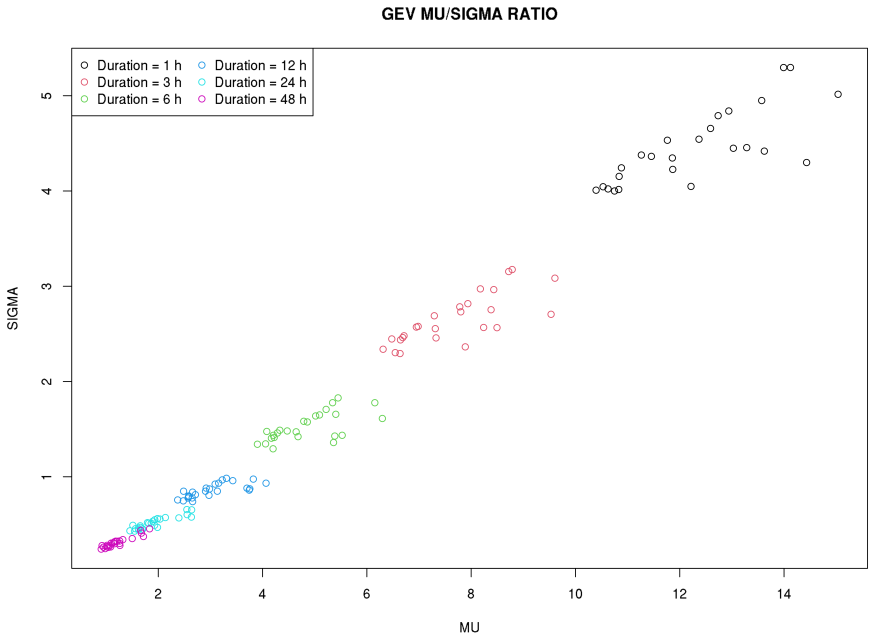

Figure S51: GEV μ/σ ratio

2.4 Widely applicable information criterion

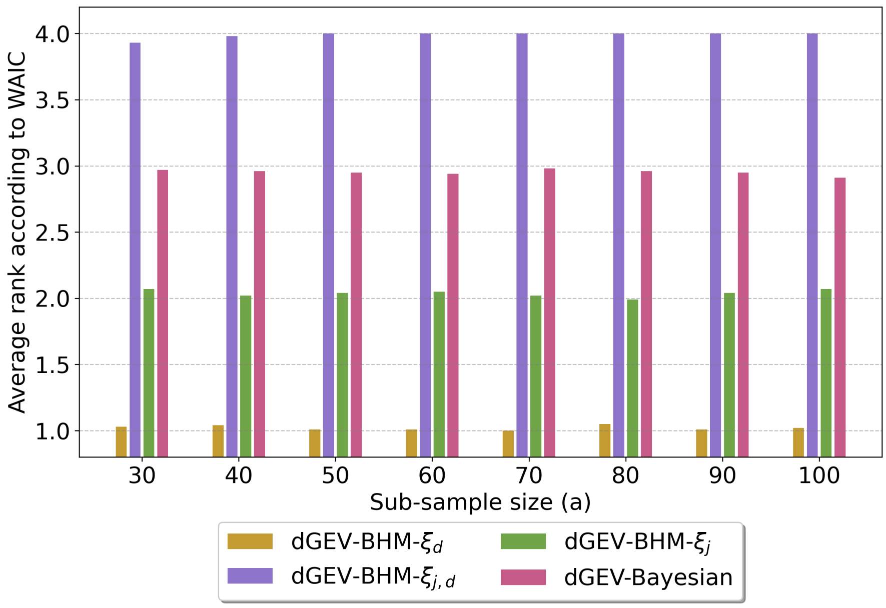

Figure S52: WAIC for all Bayesian models